Increasing/Decreasing Test

(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.

(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

proof of (a)

Let x_1 & x_2 be any numbers in the interval with $x_1 < x_2$

Goal: Show $f(x_1) < f(x_2)$.

Because $f'(x) > 0$, we know f is differentiable on $[x_1, x_2]$, by the Mean Value Theorem, there is a c between x_1 and x_2 with

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

$$\hookrightarrow f(x_2) - f(x_1) = f'(c)(x_2 - x_1)$$

But $\underset{\text{(by assumption)}}{f'(c) > 0}$ & $\underset{(b/c\ x_2 > x_1)}{x_2 - x_1 > 0}$, so

$$f(x_2)-f(x_1)=f'(c)(x_2-x_1)>0.$$

$$\square$$

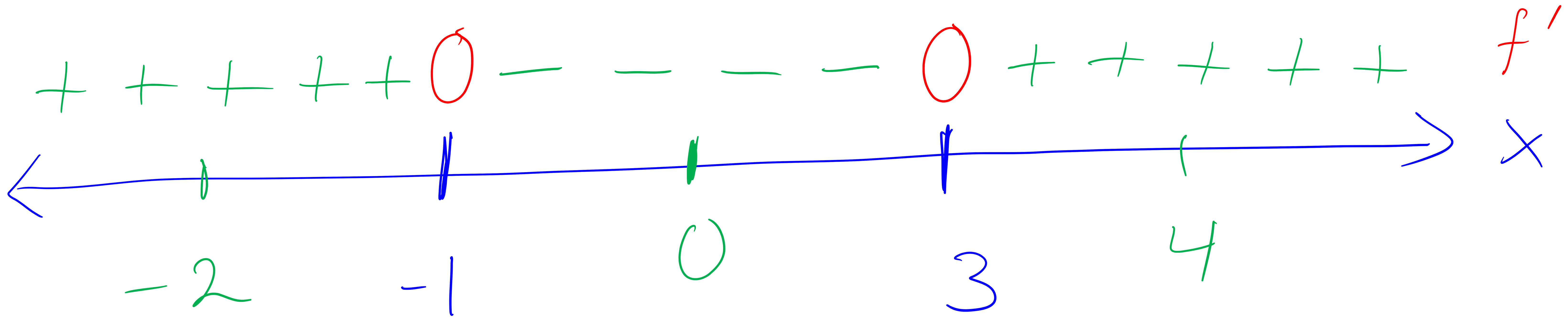
$$f(x)=x^3-3x^2-9x+4$$

Q1: Where is f increasing & decreasing?

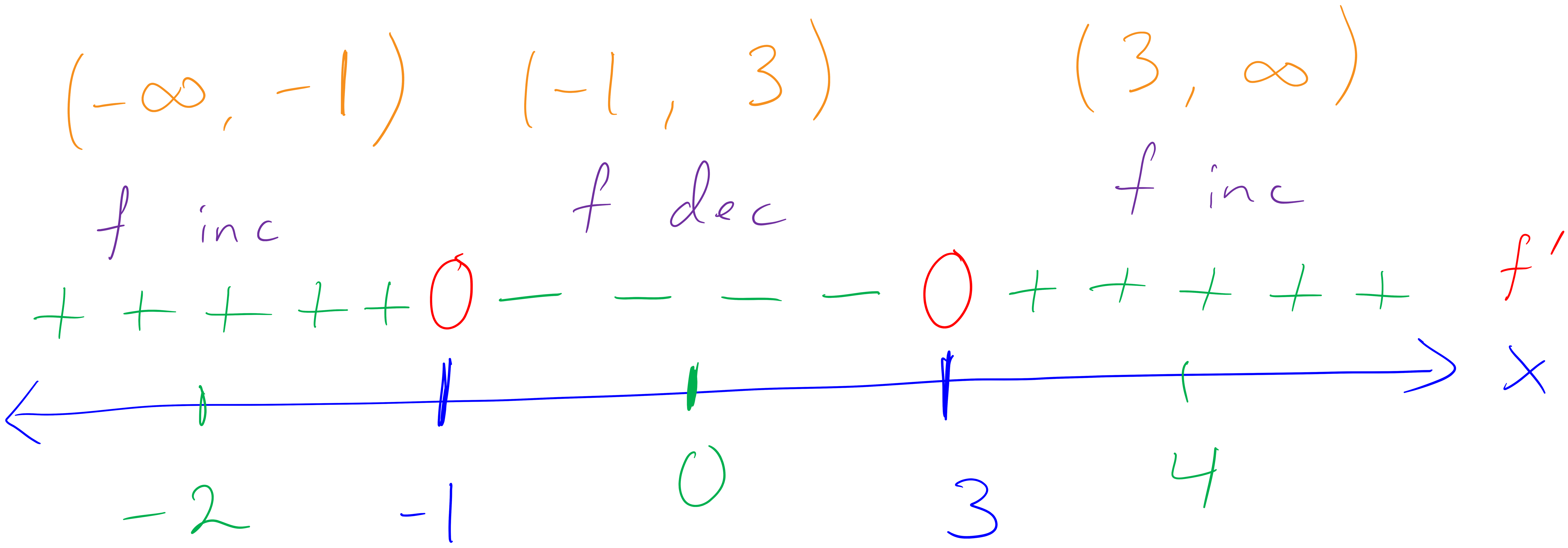
$$f'(x)=3x^2-6x-9$$

find the critical numbers

$$0=3x^2-6x-9$$
$$=3(x^2-2x-3)$$
$$=3(x-3)(x+1)\to x=3,-1$$



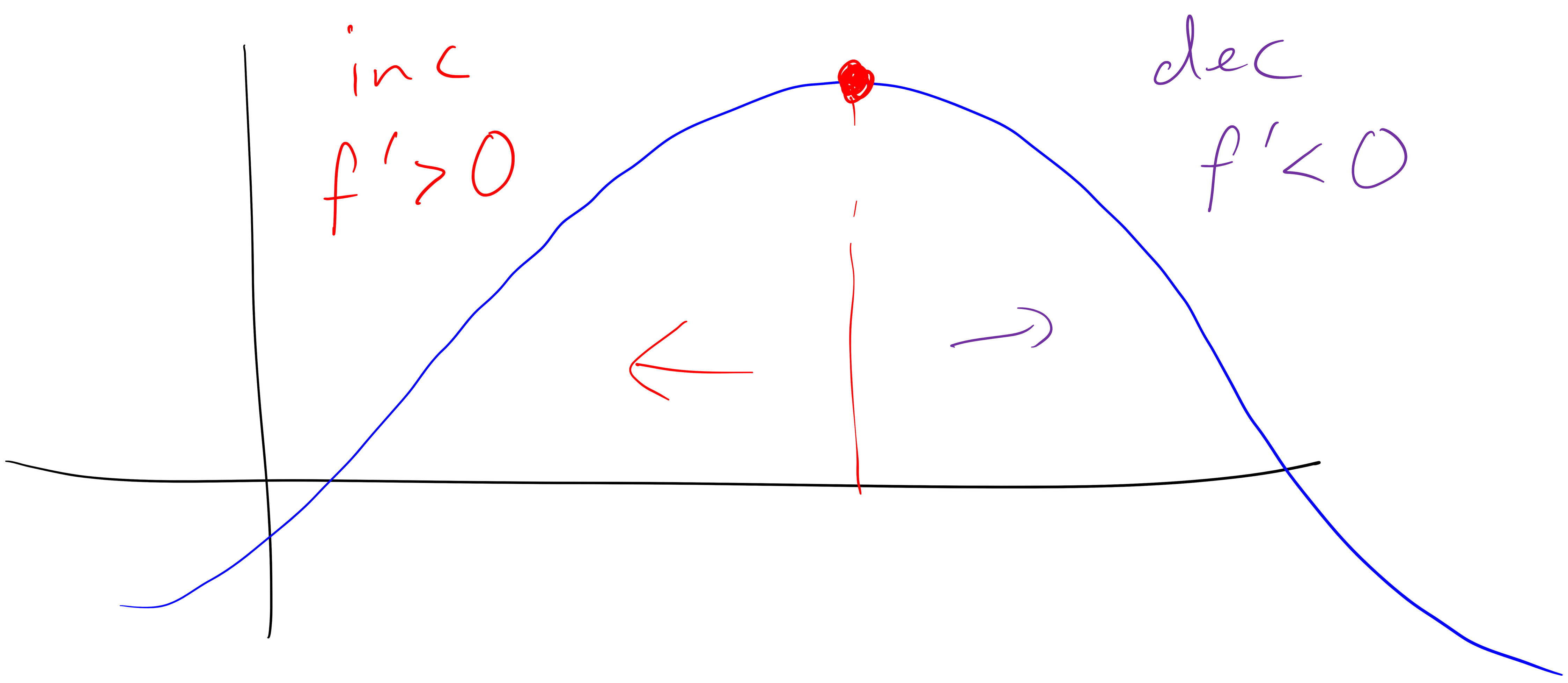
x	$3(x-3)$	$(x+1)$	$f'(x)$
-2	$-$	$-$	$+$
0	$-$	$+$	$-$
4	$+$	$+$	$+$



f is increasing on $(-\infty, -1) \cup (3, \infty)$

f is decreasing on $(-1, 3)$.

Q2: How to identify local extrema?



First Derivative Test

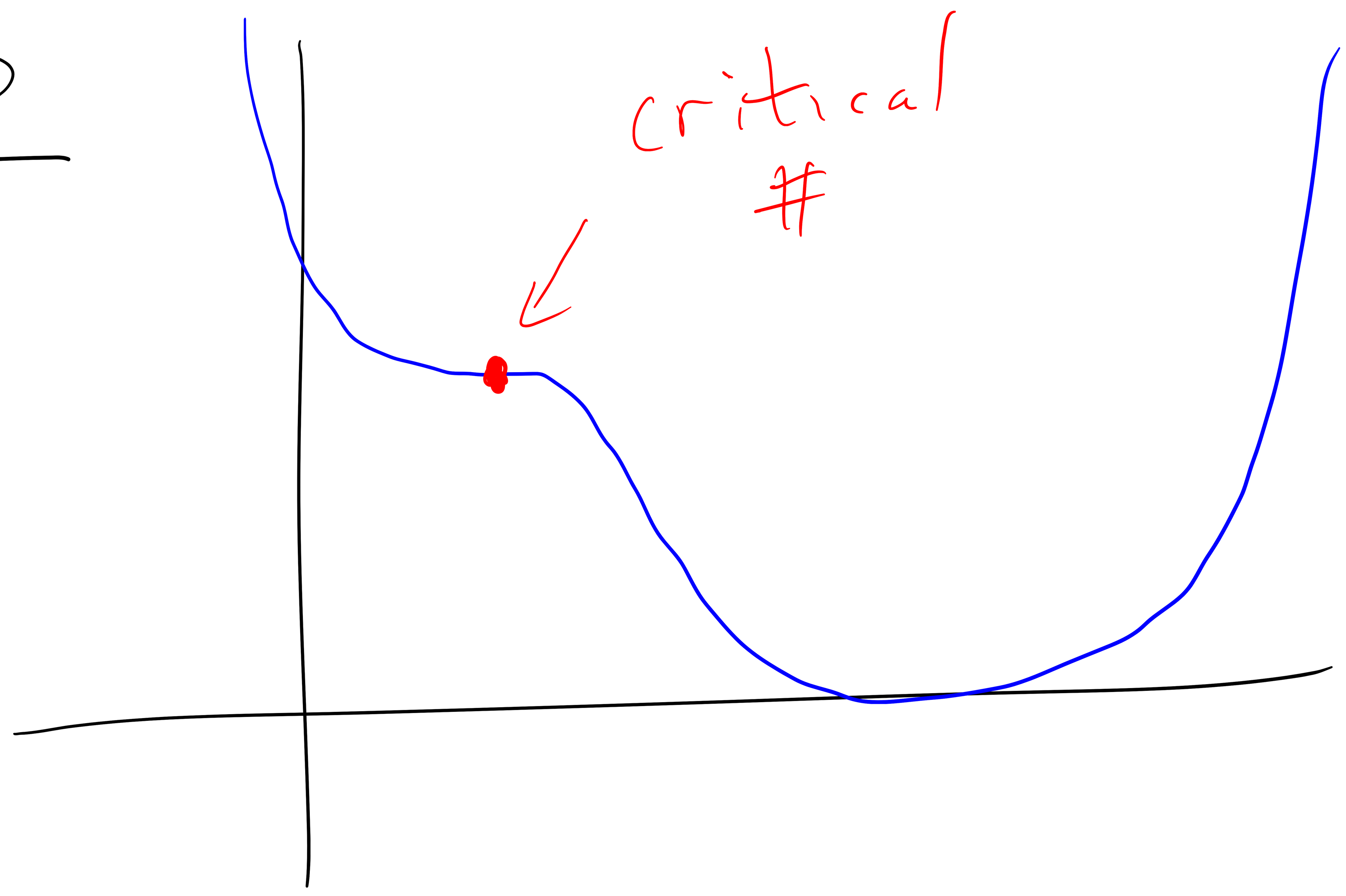
Suppose that c is a critical number of a continuous function f.

ⓐ if f' changes from positive to negative at c, then f has a local maximum at c.

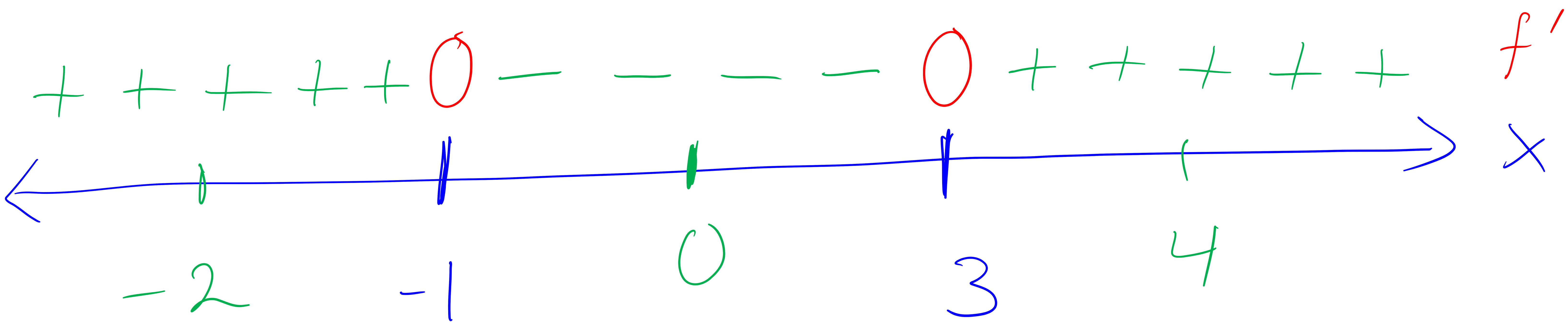
ⓑ if f' changes from negative to positive at c, then f has a local minimum at c.

ⓒ if f' does not change sign at c, then f does not have a local extremum at c.

ex of ⓒ



$$f(x) = x^3 - 3x^2 - 9x + 4$$



By the first derivative test:

f has a local maximum at $x = -1$

f has a local minimum at $x = 3$

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f has a local maximum value of $f(-1) = 9$ at $x = -1$ and a local minimum value of $f(3) = -23$ at $x = 3$

<u>Ex</u>: Find the local extrema of

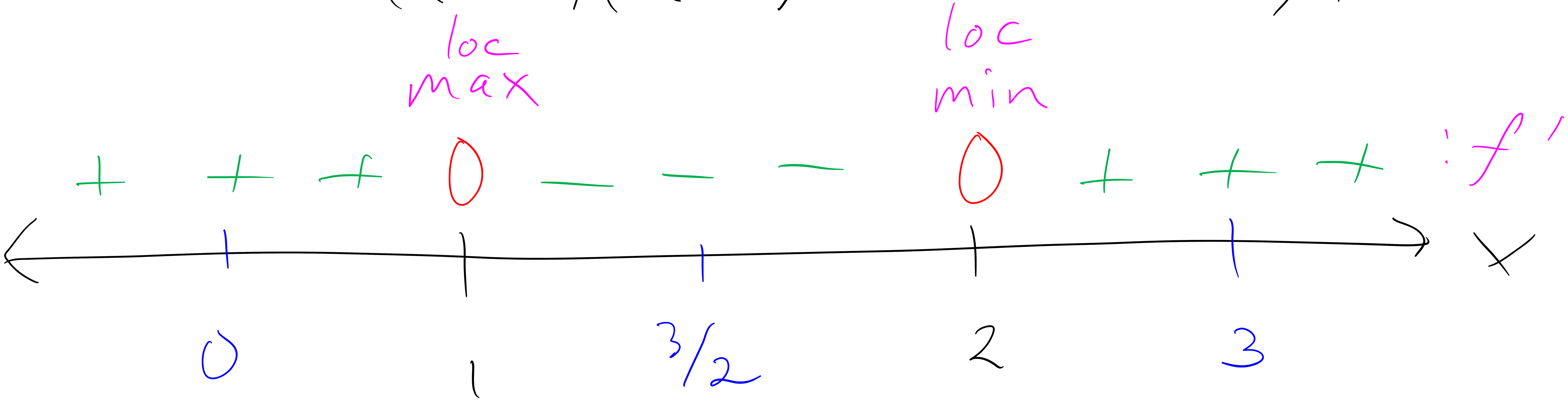
$$f(x) = 2x^3 - 9x^2 + 12x - 3.$$

<u>Sol</u>:

$$f'(x) = 6x^2 - 18x + 12$$

$$= 6(x^2 - 3x + 2)$$

$$= 6(x-2)(x-1) = 0 \longrightarrow x = 2, 1$$



x	$6(x-2)$	$(x-1)$	$f'(x)$
0	$-$	$-$	$+$
$3/2$	$-$	$+$	$-$
3	$+$	$+$	$+$